NASA TTF-10,134

EMISSIVITY OF A PINNACLED SURFACE. INVESTIGATION OF AN EMISSIVE, FIN-TUBE SYSTEM

Yu. G. Zhulev, Yu. A. Vasanov

NASA TTF-10,134

Translation of "Izluchatel'naya sposobnost' zubchatoy poverkhnosti, issledovaniye izluchatel'noy sistemy s orebrennymi trubkami".

Moscow, 1965. Source Unknown.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. MAY 1966

NASA TTF-10,134

EMISSIVITY OF A PINNACLED SURFACE. INVESTIGATION OF AN EMISSIVE, FIN-TUBE SYSTEM

Yu. G. Zhulev, Yu. A. Vasanov

27497

ABSTRACT

The article consists of two parts. /1*

The first part examines the problem of investigating the emissivity of a pinnacled surface for the case when the surrounding space is a black body with zero temperature. It is found that in the general case the effective emissivity of a pinnacled surface is a function of the degree of blackness of the lateral face of the pinnacle ε, the angle at its apex α, and the dimensionless parameter N, which depends on the following quantities: temperature of the pinnacle base, height of the pinnacle, angle at its apex, and thermal conductivity of the pinnacle material. The initial system of equations is solved on a machine for several values of ε, α, and N.

The second part examines the problem of computing the emissivity of a system made of cooled tubes which are located in parallel and which have emissive, triangular fins. The most favorable distribution of the emissive pinnacles along the cooled tube perimeter Author

* Note: Numbers in the margin indicate pagination in the original foreign text.

and the methods for optimizing the system under consideration are investigated.

I. Emissivity of a Pinnacled Surface /2

Let us investigate the two-dimensional problem of computing the emissivity of a surface formed of triangular pinnacles having a base temperature T_o and a lateral face degree of blackness ε (Figure 1). We shall first investigate pinnacles with fairly large extension, for which the law of thermal radiation and the equations of thermal conductivity along the pinnacle are valid in the following form:

$$\frac{1}{2} Q(x') = -\lambda (L' - x') \frac{\alpha}{2} \frac{dT}{dx'} \quad (1)$$

$$\frac{1}{2} dQ(x') = -\frac{q\, dx'}{\cos\frac{\alpha}{2}} \quad (2)$$

where:

Q - the thermal flux through the pinnacle cross section with the coordinate X';

λ - thermal conductivity coefficient of the pinnacle material;

α - angle at the pinnacle apex;

$q \dfrac{dx'}{\cos\frac{\alpha}{2}}$ - resulting radiation of a pinnacle lateral face element - the outgoing element of thermal flux with allowance for interirradiation of the pinnacles.

Equations (1) - (2) lead to the following expression for determining the /3 temperature distribution along a pinnacle after a change of variables.

$$(L - x) \frac{d^2T}{dx^2} - \frac{dT}{dx} - \frac{q}{\lambda \cdot \frac{\alpha}{2}} = 0; \quad (3)$$

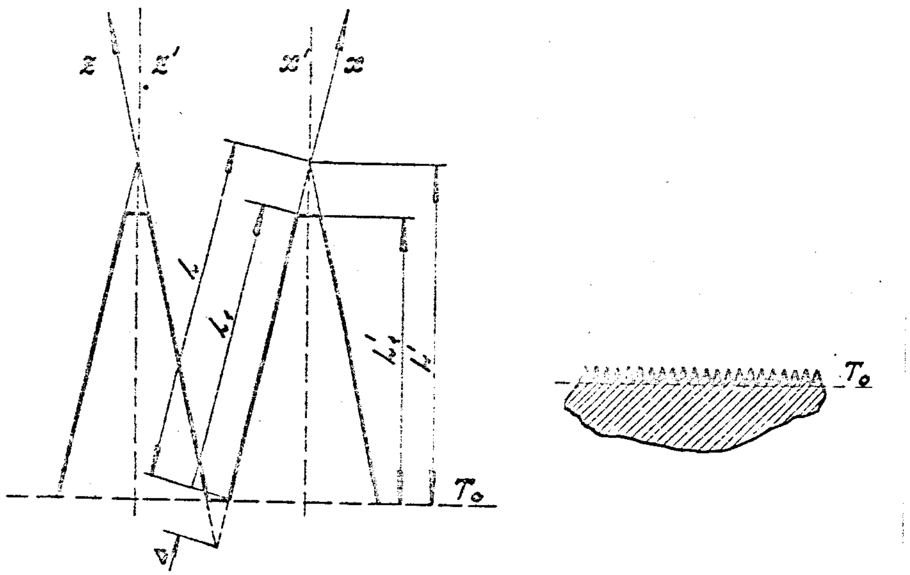


Figure 1

Let us first examine trapeziform pinnacles (Figure 1), for which the following boundary conditions of equation (3) can be assumed:

$$T = T_0 \quad \text{for} \quad x = 0;$$
$$\frac{dT}{dx} = 0 \quad \text{for} \quad x = L_1 \quad ; \qquad (4)$$

In determining q(x), we shall examine the case when the surrounding space has the parameters $\varepsilon = 1$ and T = 0. The expression for determining q(x) will then have the following form (Ref. 1)

$$q(x) = B(x) - H_{z \to x} = \varepsilon \sigma T^4(x) - \frac{\varepsilon}{2}\int_0^{L_1} \frac{B(z)(z+\Delta)(x+\Delta)\sin^2\alpha \, dz}{[(z+\Delta)^2+(x+\Delta)^2-2(x+\Delta)(z+\Delta)\cos\alpha]^{3/2}} \qquad (5)$$

where:

$H_{z \to x}$ or $H_{x \to z}$ - the specific thermal flux from the adjacent pinnacle at the pinnacle under consideration;

$B(x) = \varepsilon \sigma T^4(x) + (1-\varepsilon) H_{z \to x}$ - the specific flux of effective radiation from

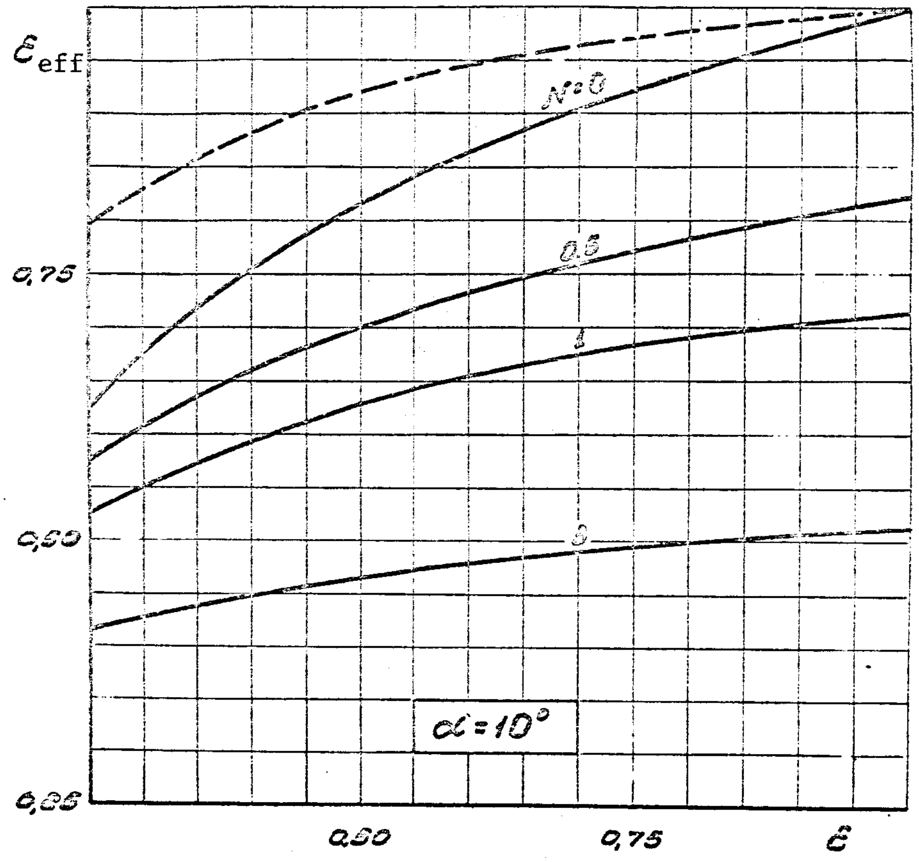


Figure 2

the surface x;

$B(z) = \varepsilon\sigma T^4(z) + (1-\varepsilon)H_{x \to z}$ – the specific flux of effective radiation from the surface z.

Let us introduce the following dimensionless variables: /4

$$\bar{T} = \frac{T}{T_0};\ \bar{x} = \frac{x}{L_1};\ \bar{z} = \frac{z}{L_1};\ \bar{L} = \frac{L}{L_1};\ \bar{H} = \frac{H}{\sigma T_0^4};\ \bar{B} = \frac{B}{\sigma T^4};\ \bar{\Delta} = \frac{\Delta}{L_1}; \quad (6)$$

In these variables, after equation (3) is integrated with the use of the second boundary condition (4), it will be written as follows:

$$(\bar{L} - \bar{x})\frac{d\bar{T}}{d\bar{x}} + N\int_{\bar{x}}^{1}[\bar{B}(\bar{z}) - \bar{H}_{z \to x}]\,d\bar{x} = 0; \quad (7)$$

where

$$\mathcal{N} = \frac{L_1 G T_0^3}{\lambda \cdot \frac{\alpha}{2}} \tag{8}$$

$$\bar{H}_{z \to x} = \frac{1}{2} \int_0^1 \frac{\bar{B}(\bar{z})(\bar{z}+\bar{\Delta})(\bar{x}+\bar{\Delta}) \sin^2\alpha \, dz}{[(\bar{z}+\bar{\Delta})^2 + (\bar{x}+\bar{\Delta})^2 - 2(\bar{x}+\bar{\Delta})(\bar{z}+\bar{\Delta})\cos\alpha]^{3/2}} \tag{9}$$

$$\bar{B}(\bar{x}) = \varepsilon \bar{T}^4(\bar{x}) + (1-\varepsilon) \bar{H}_{z \to x} \tag{10}$$

$$\bar{B}(\bar{z}) = \varepsilon \bar{T}^4(\bar{z}) + (1-\varepsilon) \bar{H}_{x \to z} \tag{11}$$

and $\overline{T} = 1$ will be the boundary conditions for equation (7) in the case of $\overline{x} = 0$.

In accordance with (5), the amount of heat radiated by one pinnacle will be written in dimensionless variables as follows:

$$q_\Sigma = 2 L_1 \cdot G \cdot T_0^4 \int_0^1 [\bar{B}(\bar{x}) - \bar{H}_{z \to x}] \, d\bar{x} \tag{12}$$

The effective degree of blackness of a pinnacled surface will be determined as the ratio of the heat radiated by the pinnacle or by the trough q_Σ to the heat which would be radiated by the pinnacle base having the temperature T_0 and the degree of blackness $\varepsilon = 1$ ($q_{id} = 2L_1GT_0^4 \sin\frac{\alpha}{2}$). Therefore, the effective degree of blackness for the surface under consideration will be determined by the following expression

$$\varepsilon_{\text{eff}} = \frac{1}{\sin\frac{\alpha}{2}} \int_0^1 [\bar{B}(\bar{x}) - \bar{H}_{z \to x}] \, d\bar{x} \tag{13}$$

from which it can be seen that it is a function of the parameters

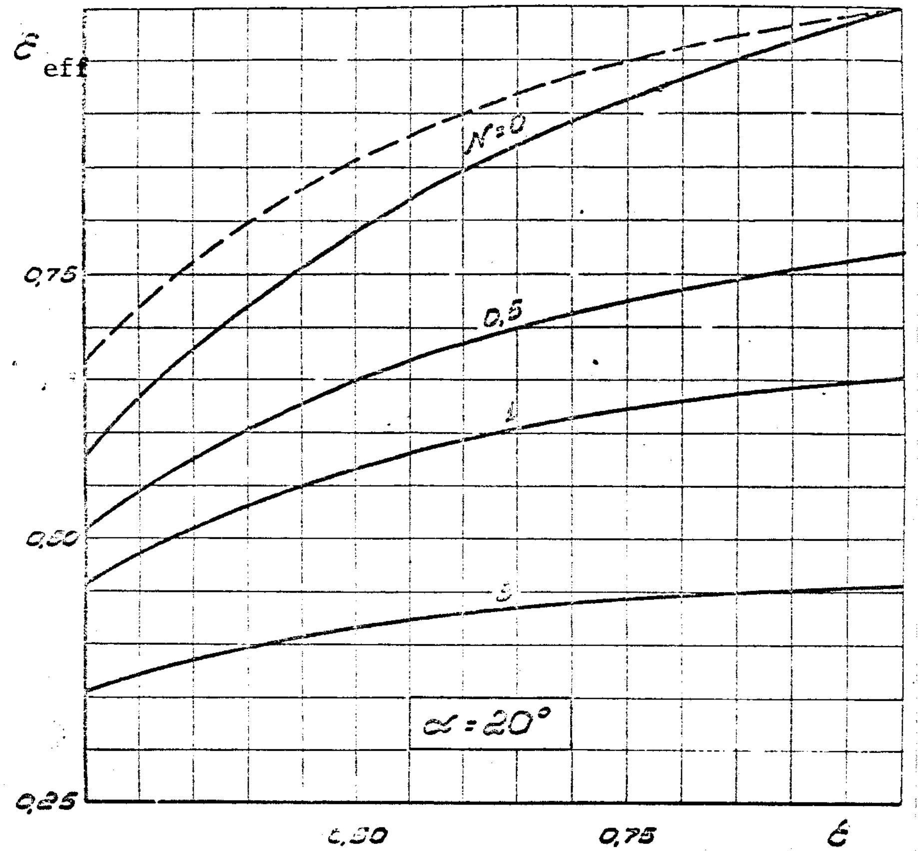


Figure 3

N, α and ε

The system (7)-(11) and (13) was solved on an electronic computer by numerical method, and the solution was of an asymptotic type in the case of $\bar{L} \to 1$ and $\bar{\Lambda} \to 0$.

Figures 2, 3 and 4 show the dependence of the effective degree of blackness of a pinnacled surface on the degree of blackness ε of a surface with pinnacles * for different values of the thermal conductivity parameter N and the angle α. It can be seen that for small values of ε the pinnacled surface can sharply increase the effective degree of blackness in the case of N → 0, which is significant in the case of high temperatures at which it may be difficult to use coatings which increase the degree of blackness. On the

*Translator's Note: Based on the context, this should correctly read "without pinnacles".

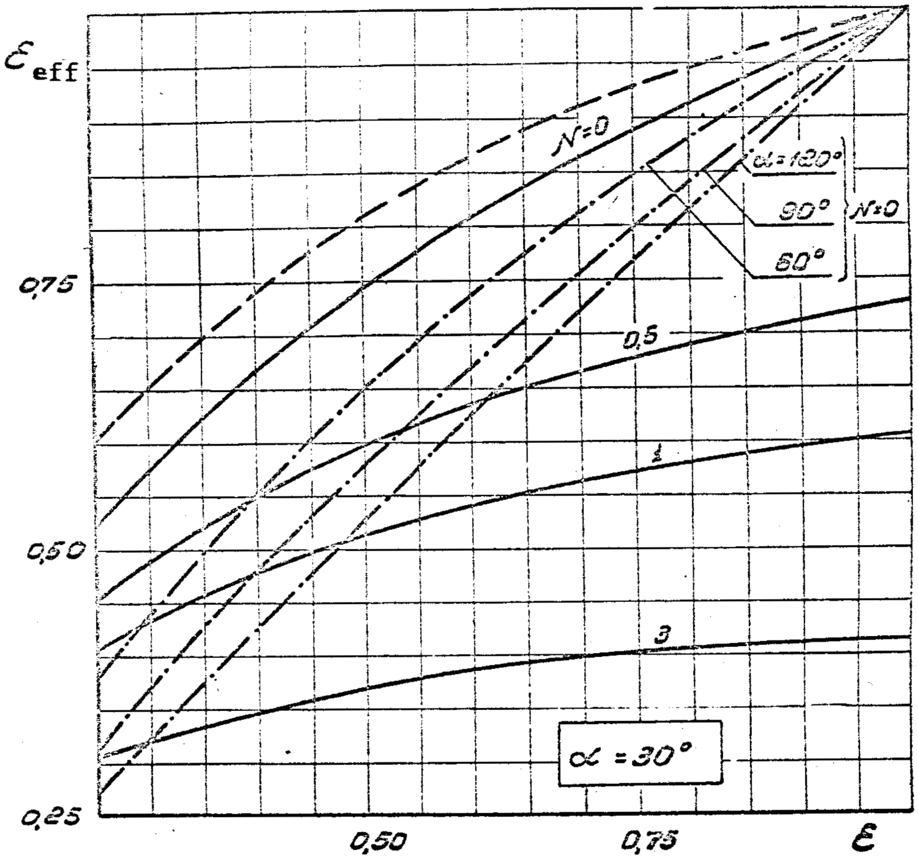


Figure 4

other hand, when necessary one can significantly lower the effective degree of blackness by providing rather large values of the parameter N.

The nature of the relationships in Figures 2, 3 and 4 can be explained /6 by the fact that not only the geometry of the pinnacles and the degree of blackness of their surfaces, but also the temperature decrease occurring between its base and the apex (the larger the parameter N, the greater the temperature decrease along the pinnacle) can influence the respective emissivity of the pinnacled surface.

The approximate solution of the problem regarding the effective emissivity of an isothermal pinnacled surface, which corresponds to a particular case of the problem under consideration when $N = 0$, which was obtained in (Ref. 2), is plotted in Figures 2-4. For this case, the dependences obtained above are valid for any values of α, and therefore for $N = 0$ computations were also performed for $\alpha = 60$; 90 and 120° (the dot-dash lines in Figure 4, which practically coincide with those obtained in [Ref. 2] for $\alpha \geqslant 90°$, are approximate relationships).

The results obtained can be applied to computing the emissivity of the pinnacled surface of a cylinder. In this case, in Figures 2-4 the angle α does not designate the angle at the pinnacle apex, but rather the angle between the lateral faces of the pinnacles, which equals $\alpha + \frac{2\pi}{\eta}$, where η is the number of pinnacles on the cylinder.

2. Investigation of an Emissive, Fin-Tube System

Let us investigate the two-dimensional problem of computing the thermal radiation of an infinite system of star-shaped emitters having four points with triangular heat transfer fins; these fins are located at the apexes of a cooled prism and are distributed symmetrically with respect to the plane containing the axis of the individual emitters (Figure 5a). This system corresponds quite accurately to a system of cooled tubes which are located in parallel and which have emitting triangular fins.

We shall solve this problem under the following assumptions:

1. The heat transfer fins of the emitters have the same geometry.

2. The physical thermal properties of the fin material do not depend on temperature.

3. The temperature of a side of the cooled prism is constant along the perimeter.

4. The surrounding space is a black body with zero temperature.

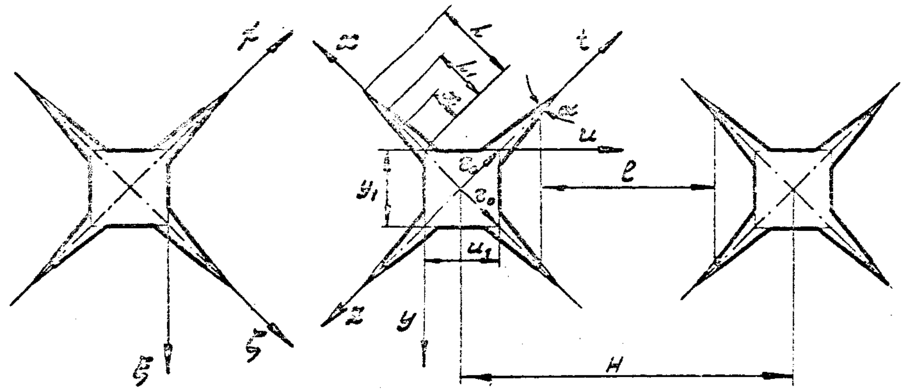


Figure 5a

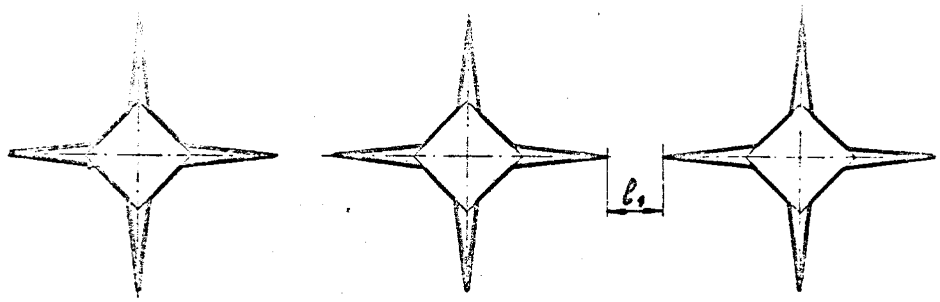


Figure 5b

5. The surfaces of the fins and the sides are gray diffusive emitters.

6. The emissive fins are fairly thin, so that the transverse temperature gradient in the fin can be disregarded, as compared with the longitudinal gradient.

Taking the above assumptions into account, we can write the law for thermal radiation and the equation of heat conductivity along the fin in the following way: /8

$$Q(x) = -2\lambda(l - x)\frac{\delta}{2}\cdot\frac{dT}{dx} \tag{14}$$

$$dQ(x) = -[q_1(x) + q_2(x)]\,dx \tag{15}$$

where λ - the heat conductivity coefficient of the finned material,

α - the angle between the fin lateral faces,

L - height of the fin,

Q(x) - **thermal flux** through the fin cross section with the coordinate x,

T - fin temperature,

$q_1(x)$ - specific flux of resulting radiation from the fin surface on which thermal fluxes fall, not only from the adjacent fin and the prism side, but also from the elements of the adjacent star-shaped emitter (Figure 5a),

$q_2(x)$ - specific flux of resulting radiation from the fin surface which is in a state of radient heat exchange only with the adjacent fin and the prism side.

Equations (14) and (15) lead to the following expression for determining the temperature distribution along the fin /9

$$(L - x)\frac{d^2T}{dx^2} - \frac{dT}{dx} - \frac{q_1(x) + q_2(x)}{\lambda \cdot \frac{\alpha}{2}} = 0 \tag{16}$$

Let us first examine trapeziform fins having the length L_1 (Figure 5a). In this case, we can assume the following boundary conditions for equation (16)

$$T = T_0 \quad \text{for} \quad x = 0 \text{ and } \frac{dT}{dx} = 0 \quad \text{for} \quad x = L_1 \tag{17}$$

where T_0 is the side temperature of the cooled prism.

The specific fluxes of the resulting radiation from the fin surfaces will be determined by the following expressions:

$$q_1(x) = B(x) - \left[H_{y\to x}(x) + H_{z\to x}(x) + H_{L\to x}(x) + H_{\xi\to x}(x) + H_{\zeta\to x}(x)\right] \tag{18}$$

$$q_2(x) = B^*(x) - [H_{u\to x}(x) + H_{t\to x}(x)] \tag{19}$$

where

$$B(x) = \varepsilon\sigma T^4(x) + (1-\varepsilon)[H_{y\to x}(x) + H_{z\to x}(x) +$$

$$+ H_{l\to x}(x) + H_{\xi\to x}(x) + H_{\zeta\to x}(x)] \tag{20}$$

- the specific flux of effective radiation from the fin surface under consideration on which thermal fluxes fall, not only from the adjacent fin and the prism side, but also from elements of the adjacent star-shaped emitter /10 (in equation (20) on, ε is the degree of blackness of the emitter surface, and σ is the Stefan-Boltzmann constant)

$$B^*(x) = \varepsilon\sigma T^4(x) + (1-\varepsilon)[H_{u\to x}(x) + H_{t\to x}(x)] \tag{21}$$

- the specific flux of effective radiation from the fin surface which is in a state of radiant heat exchange only with the adjacent fin and the prism side;

$$H_{y\to x}(x);\ H_{z\to x}(x);\ H_{l\to x}(x);\ H_{\xi\to x}(x);\ H_{\zeta\to x}(x);$$

$$H_{u\to x}(x);\ H_{t\to x}(x);$$

- the specific fluxes on the fin under consideration from surfaces designated by the first letter of the subscript.

Assuming that the emitting element of the fin surface is located in the fin plane of symmetry, we can obtain the following relationships for determining specific fluxes on the fin under consideration (Ref. 1):

$$H_{y\to x}(x) = \frac{1}{4}\int_0^{y_1} B(y)\frac{xy\,dy}{(x^2+y^2+xy\sqrt{2})^{3/2}}\ ; \tag{22}$$

$$H_{z\to x}(x)=\frac{1}{2}\int_0^{l_1} B(z)\,\frac{(z_0+z)(z_0+x)\,dz}{[(z_0+z)^2+(z_0+x)^2]^{3/2}}\;; \tag{23}$$

$$H_{\chi\to x}(x)=\frac{1}{2}\int_0^{l_1} B(\chi)\,\frac{(l_1+\frac{\ell}{\sqrt{2}}-x)(l_1+\frac{\ell}{\sqrt{2}}-\chi)\,d\chi}{[(l_1+\frac{\ell}{\sqrt{2}}-x)^2+(l_1+\frac{\ell}{\sqrt{2}}-\chi)^2]^{3/2}}\;; \tag{24}$$

$$H_{\xi\to x}(x)=\frac{1}{2}\int_0^{\xi_1} B(\xi)\,\frac{(2l_1-x+\ell\sqrt{2})(2l_1+\ell\sqrt{2}+\xi\sqrt{2})\,d\xi}{[(2l_1-x+\ell\sqrt{2})+(x+\xi\sqrt{2})^2]^{3/2}}\;; \tag{25}$$ /11

$$H_{\zeta\to x}(x)=\frac{1}{2}\int_0^{l_1} B(\zeta)\,\frac{(z_0+l_1+\frac{\ell}{\sqrt{2}})^2\,d\zeta}{\{(z_0+l_1+\frac{\ell}{\sqrt{2}})^2+[(l_1+\frac{\ell}{\sqrt{2}}-z_0)+(x+\zeta)]^2\}^{3/2}} \tag{26}$$

$$H_{u\to x}(x)=\frac{1}{4}\int_0^{u_1} B(u)\,\frac{ux\,du}{(u^2+x^2+ux\sqrt{2})^{3/2}}\;; \tag{27}$$

$$H_{t\to x}(x)=\frac{1}{2}\int_0^{l_1} B^*(t)\,\frac{(z_0+x)(z_0+t)\,dt}{[(z_0+x)^2+(z_0+x)^2]^{3/2}}\;; \tag{28}$$

where z_0 is the radius of the circle circumscribing the prism (see Figure 5a);

ℓ - the distance between the fin ends of adjacent emitters (see Figure 1a); $y_1=\xi_1=u_1$ - dimension of the prism sides;

$B(z); B(y); B(\zeta); B(\xi); B(\chi); B(u); B^x(t)$ - specific fluxes of effective radiation from surfaces designated by the index in the parenthesis.

The expressions for determining the specific fluxes of effective radiation included in formulas (22)-(28) have the following form:

$$B(y)=\varepsilon\sigma T_0^4+(1-\varepsilon)[H_{x\to y}(y)+H_{z\to y}(y)+H_{\zeta\to y}(y)+H_{\xi\to y}(y)+H_{\chi\to y}(y)] \tag{29}$$

$$B(u) = \varepsilon \sigma T_0^4 + (1-\varepsilon)[H_{x\to u}(u) + H_{t\to u}(u)]; \tag{30}$$

$$B(z) = B(\zeta) = B(\chi) = B(x); \quad B(\xi) = B(y); \quad B^*(t) = B^*(x) \tag{31}$$

where the specific fluxes $H_{x\to y}(y);\ H_{z\to y}(y);\ H_{\zeta\to y}(y);$ /12

$H_{\xi\to y}(y);\ H_{\chi\to y}(y);\ H_{x\to u}(u);\ H_{t\to u}(u)$ are determined by means of the following relationships (Ref. 1):

$$H_{x\to y}(y) = \frac{1}{4}\int_0^{L_1} B(x)\frac{xy\,dx}{(x^2+y^2+xy\sqrt{2})^{3/2}}; \tag{32}$$

$$H_{z\to y}(y) = \frac{1}{4}\int_0^{L_1} B(z)\frac{z(y_1-y)\,dz}{[z^2+(y_1-y)^2+z(y_1-y)\sqrt{2}]^{3/2}} \tag{33}$$

$$H_{\zeta\to y}(y) = \frac{1}{2}\int_0^{L_1} B(\zeta)\frac{(2L_1+\ell\sqrt{2}-\zeta)[2L_1+\ell\sqrt{2}+(y_1-y)\sqrt{2}]\,d\zeta}{\{[\zeta+(y_1-y)\sqrt{2}]^2+(2L_1+\ell\sqrt{2}-\zeta)^2\}^{3/2}} \tag{34}$$

$$H_{\xi\to y}(y) = \frac{1}{2}\int_0^{\xi_1} B(\xi)\frac{(\ell+L_1\sqrt{2})^2\,d\xi}{[(y-\xi)^2+(\ell+L_1\sqrt{2})^2]^{3/2}} \tag{35}$$

$$H_{\chi\to y}(y) = \frac{1}{2}\int_0^{L_1} B(\chi)\frac{(2L_1+\ell\sqrt{2}-\chi)(2L_1+\ell\sqrt{2}+y\sqrt{2})\,d\chi}{[(\chi-y\sqrt{2})^2+(2L_1+\ell\sqrt{2}-\chi)^2]^{3/2}} \tag{36}$$

$$H_{x\to u}(u) = \frac{1}{4}\int_0^{L_1} B^*(x)\frac{xu\cdot dx}{(x^2+u^2+xu\sqrt{2})^{3/2}} \tag{37}$$

$$H_{t\to u}(u) = \frac{1}{4}\int_0^{L_1} B^*(t)\frac{t(u_1-u)\,dt}{[t^2+(u_1-u)^2+t(u_1-u)\sqrt{2}]^{3/2}} \tag{38}$$

An analysis of the relationships obtained shows that there are 19 unknown functions, taking into account equation (18):

$$\bar{T}(\bar{x}),\ \bar{B}(\bar{x}),\ \bar{B}^{*}(\bar{x}),\ \bar{B}(\bar{y}),\ \bar{B}(\bar{u}),\ \bar{H}_{y\to x}(\bar{x}),\ \bar{H}_{z\to x}(\bar{x}),\ \bar{H}_{\chi\to x}(\bar{x}),$$
$$\bar{H}_{\xi\to x}(\bar{x}),\ \bar{H}_{\varsigma\to x}(\bar{x}),\ \bar{H}_{u\to x}(\bar{x}),\ \bar{H}_{t\to x}(\bar{x}),\ \bar{H}_{x\to y}(\bar{y}),\ \bar{H}_{z\to y}(\bar{y}),\ \bar{H}_{\xi\to y}(\bar{y}),$$
$$\bar{H}_{\varsigma\to y}(\bar{y}),\ \bar{H}_{\chi\to y}(\bar{y}),\ \bar{H}_{x\to u}(\bar{u}),\ \bar{H}_{t\to u}(\bar{u})$$

In order to determine these 19 unknown functions, we have a system of 19 /13 equations: equations (16) and (22)-(38).

Let us introduce the following dimensionless variables:

$$\bar{T}=\frac{T}{T_0};\ \bar{H}=\frac{H}{\sigma T_0^4};\ \bar{B}=\frac{B}{\sigma T_0^4};\ \bar{z}_0=\frac{z}{L_1};\ \bar{l}=\frac{l}{L_1};$$
$$\bar{L}=\frac{L}{L_1};\ \bar{x}=\frac{x}{L_1};\ \bar{y}=\frac{y}{L_1};\ \bar{z}=\frac{z}{L_1};\ \bar{u}=\frac{u}{L_1};\ \bar{t}=\frac{t}{L_1}; \tag{39}$$
$$\bar{\chi}=\frac{\chi}{L_1};\ \bar{\xi}=\frac{\xi}{L_1};\ \bar{\varsigma}=\frac{\varsigma}{L_1};$$

In these variables, after equation (16) is integrated with the use of the second boundary condition (4), it is written in the following way with allowance for relationships (18) and (19):

$$(\bar{L}-\bar{x})\frac{d\bar{T}}{d\bar{x}}+N_1\int_{\bar{x}}^{1}\Big\{\big[\bar{B}(\bar{x})+\bar{B}^{*}(\bar{x})\big]-\big[\bar{H}_{y\to x}(\bar{x})+\bar{H}_{z\to x}(\bar{x})+$$
$$+\bar{H}_{\chi\to x}(\bar{x})+\bar{H}_{\xi\to x}(\bar{x})+\bar{H}_{\varsigma\to x}(\bar{x})+\bar{H}_{u\to x}(\bar{x})+\bar{H}_{t\to x}(\bar{x})\big]\Big\}d\bar{x}=0 \tag{40}$$

where

$$N_1=\frac{L_1\,\sigma\cdot T_0^{3}}{\lambda\,\alpha} \tag{41}$$

and the remaining 18 equations in dimensionless variables can be obtained from relationships (22) - (38), in which the real variables are replaced by the dimensionless variables introduced.

$$\bar{B}(\bar{x}) = \varepsilon \bar{T}^4(\bar{x}) + (1-\varepsilon)\left[\bar{H}_{y\to x}(\bar{x}) + \bar{H}_{z\to x}(\bar{x}) + \bar{H}_{\chi\to x}(\bar{x}) + \bar{H}_{\xi\to x}(\bar{x}) + \bar{H}_{\varsigma\to x}(\bar{x})\right]$$ /14

$$\bar{B}^*(\bar{x}) = \varepsilon \bar{T}^4(\bar{x}) + (1-\varepsilon)\left[\bar{H}_{u\to x}(\bar{x}) + \bar{H}_{t\to x}(\bar{x})\right]$$

$$\bar{H}_{z\to x}(\bar{x}) = \frac{1}{2}\int_0^1 \bar{B}(\bar{z}) \frac{(\bar{z}_0+\bar{z})(\bar{z}_0+\bar{x})\,d\bar{z}}{\left[(\bar{z}_0+\bar{z})^2+(\bar{z}_0+\bar{x})^2\right]^{3/2}}$$

$$\bar{H}_{y\to x}(\bar{x}) = \frac{1}{4}\int_0^{\bar{y}_1} \bar{B}(\bar{y}) \frac{\bar{x}\bar{y}\,d\bar{y}}{(\bar{x}^2+\bar{y}^2+\bar{x}\bar{y}\sqrt{2})^{3/2}}\ ; \text{ etc.}$$

In accordance with (18) and (19), the amount of heat emitted by a unit of fin **length** (fin length designates the fin extension in a direction perpendicular to the plane of the drawing in Figure 5a) can be written in dimensionless variables as follows:

$$q_p = L_1 \sigma T_0^4 \int_0^1 \Big\{\left[\bar{B}(\bar{x}) + \bar{B}^*(\bar{x})\right] - \left[\bar{H}_{y\to x}(\bar{x}) + \bar{H}_{z\to x}(\bar{x}) + \right. \tag{42}$$

$$\left. + \bar{H}_{\chi\to x}(\bar{x}) + \bar{H}_{\xi\to x}(\bar{x}) + \bar{H}_{\varsigma\to x}(\bar{x}) + \bar{H}_{u\to x}(\bar{x}) + \bar{H}_{t\to x}(\bar{x})\right]\Big\}\, d\bar{x}$$

The amount of heat emitted by a unit of prism side length will be determined from the following relationships:

(a) For that prism side which is in a state of radiant heat exchange with the adjacent emitter:

$$q_{pr\,1} = L_1 \sigma T_0^4 \int_0^{\bar{y}_1} \Big\{\bar{B}(\bar{y}) - \left[\bar{H}_{x\to y}(\bar{y}) + \bar{H}_{z\to y}(\bar{y}) + \bar{H}_{\chi\to y}(\bar{y}) + \right. \tag{43}$$

$$\left. + \bar{H}_{\xi\to y}(\bar{y}) + \bar{H}_{\varsigma\to y}(\bar{y})\right]\Big\}\, d\bar{y}$$

(b) For that prism side which is not in a state of radiant heat /15

exchange with the adjacent emitter

$$q_{pr.2} = L_1 \sigma T_0^4 \int_0^{\bar{u}_1} \{ \bar{B}(\bar{u}) - [\bar{H}_{x\to u}(\bar{u}) + \bar{H}_{t\to u}(\bar{u})] \} d\bar{u} \tag{44}$$

Let us now determine the emission coefficient of a system which represents the ratio of heat emitted by the emitter under consideration $2(2q_p + q_{pr.1} + q_{pr.2})$ to the heat q_{id} which would be emitted by the given emitter in the case of infinitely large thermal conductivity of the fin material, $\ell = \infty$ and $\varepsilon = 1$. The expression for determining q_{id} will have the following form:

$$q_{id} = 8(L_1 + e_0)\sigma T_0^4 \sin 45^\circ \tag{45}$$

and the emission coefficient will be determined by the following expression:

$$\begin{aligned}\eta = \frac{1}{4(1-\bar{e}_0)\sin 45^\circ} \Bigg\{ \Bigg\{ 2\int_0^1 \Big\{ [\bar{B}(\bar{x}) + \bar{B}^x(\bar{x})] - [\bar{H}_{y\to x}(\bar{x}) + \bar{H}_{z\to x}(\bar{x}) + \\ + \bar{H}_{\chi\to x}(\bar{x}) + \bar{H}_{\xi\to x}(\bar{x}) + \bar{H}_{\varsigma\to x}(\bar{x}) + \bar{H}_{u\to x}(\bar{x}) + \bar{H}_{t\to x}(\bar{x})] \Big\} d\bar{x} + \\ + \int_0^{\bar{y}_1} \{ \bar{B}(\bar{y}) - [\bar{H}_{x\to y}(\bar{y}) + \bar{H}_{z\to y}(\bar{y}) + \bar{H}_{\chi\to y}(\bar{y}) + \bar{H}_{\xi\to y}(\bar{y}) + \bar{H}_{\varsigma\to y}(\bar{y})] \} dy + \\ + \int_0^{\bar{u}_1} \{ \bar{B}(\bar{u}) - [\bar{H}_{x\to u}(\bar{u}) + \bar{H}_{t\to u}(\bar{u})] \} d\bar{u} \Bigg\} \Bigg\}\end{aligned} \tag{46}$$

from which it can be seen that it is a function of the parameters /16

N_1; $\bar{e}_0$; $\mathcal{E}$ and $\bar{\mathcal{E}}$. It can be determined by solving equation (40) together with equations (22) - (38) written in dimensionless variables.

In the case of the specific dependence $\eta = \eta(N, \bar{e}_0, \mathcal{E}, \bar{\mathcal{E}})$ and

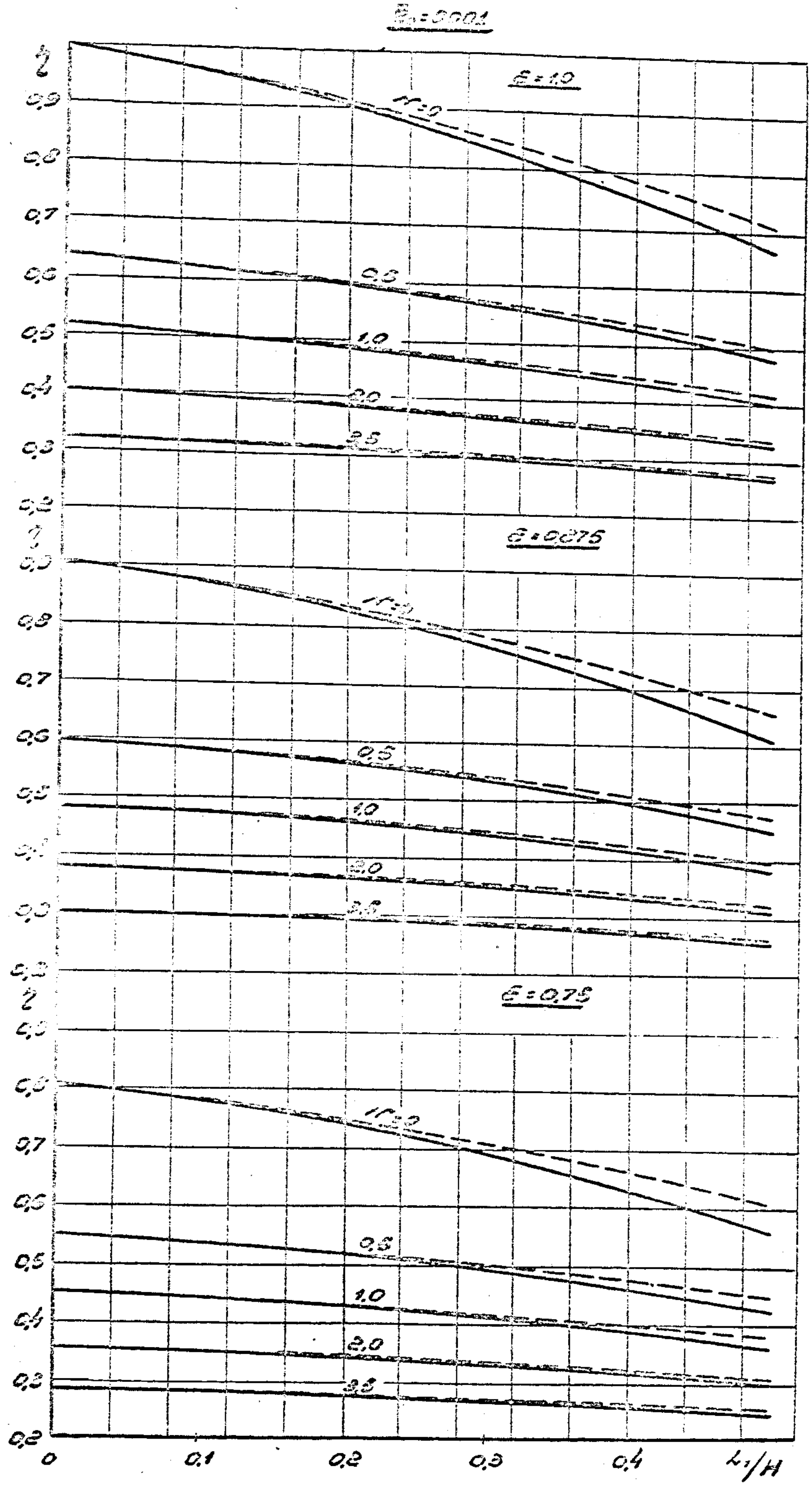


Figure 6

/23

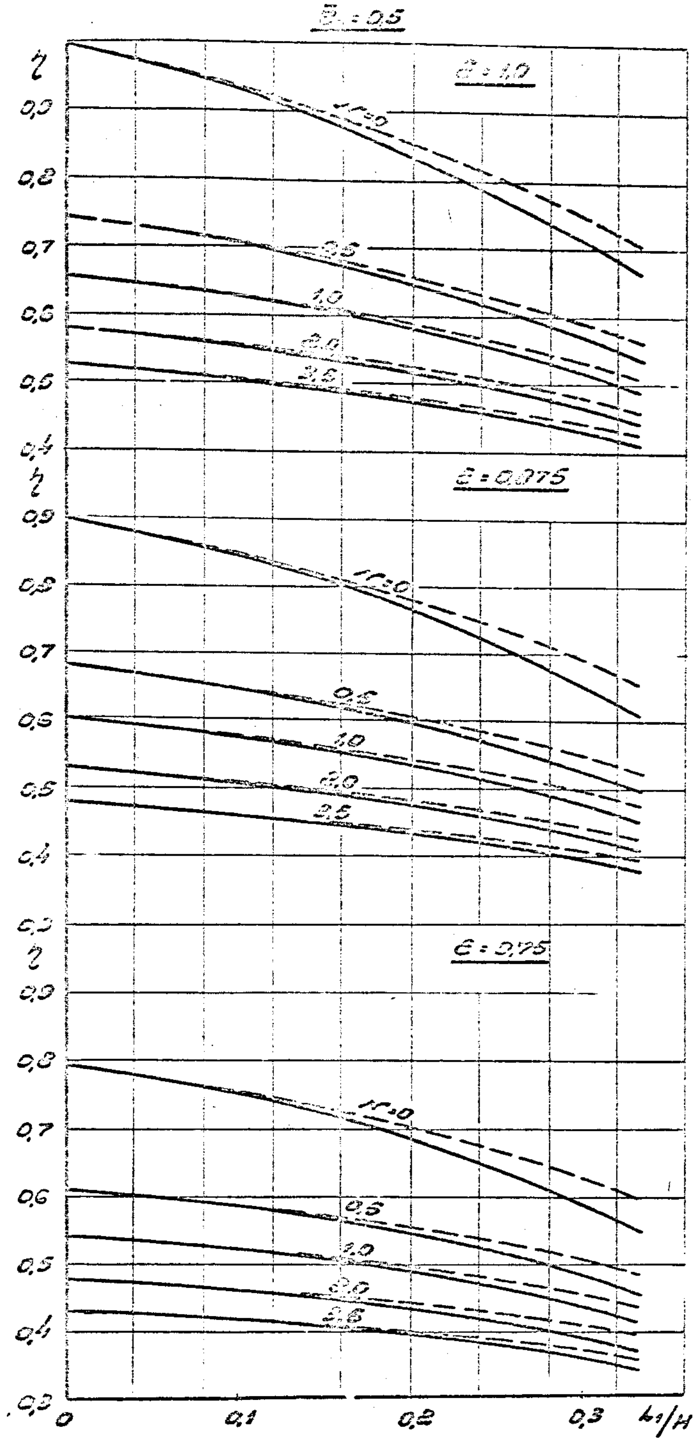
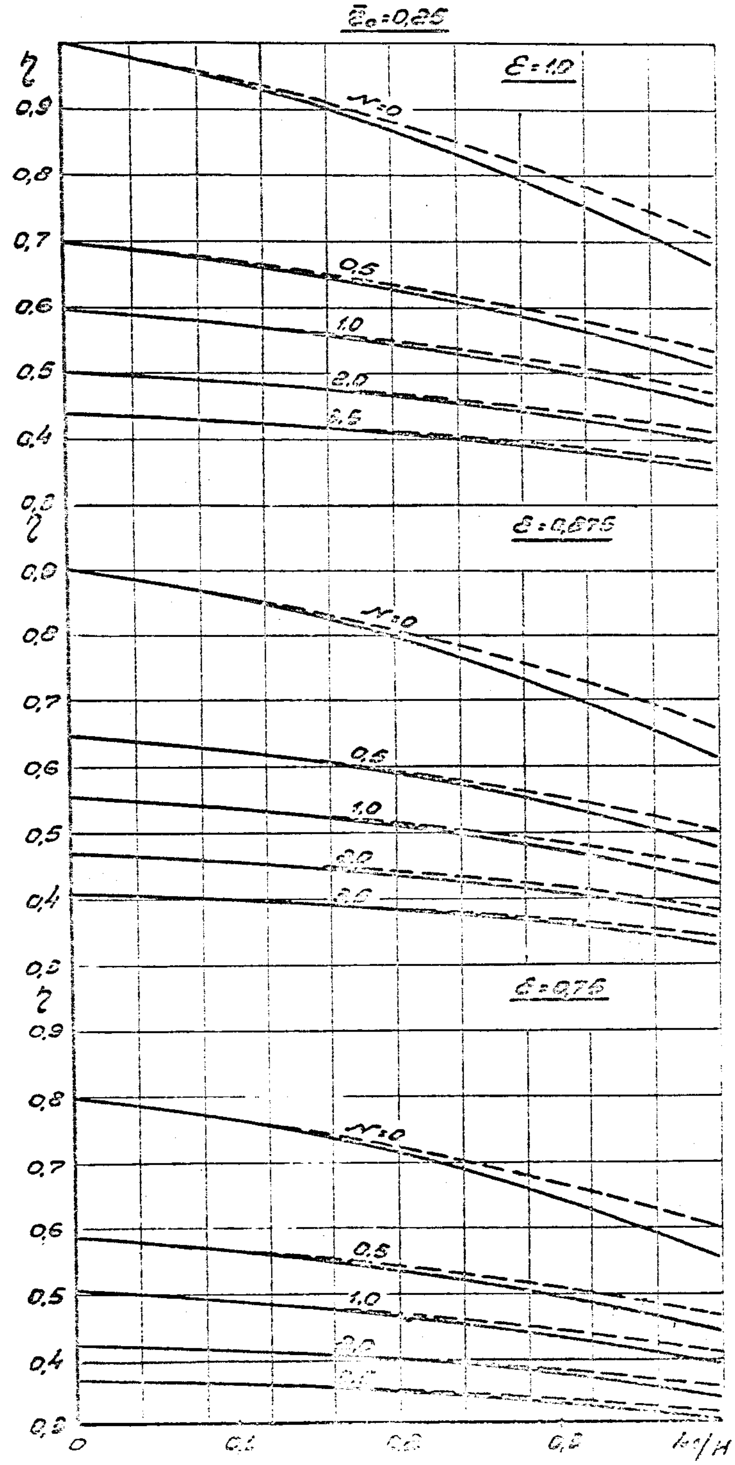


Figure 7



/24

Figure 8

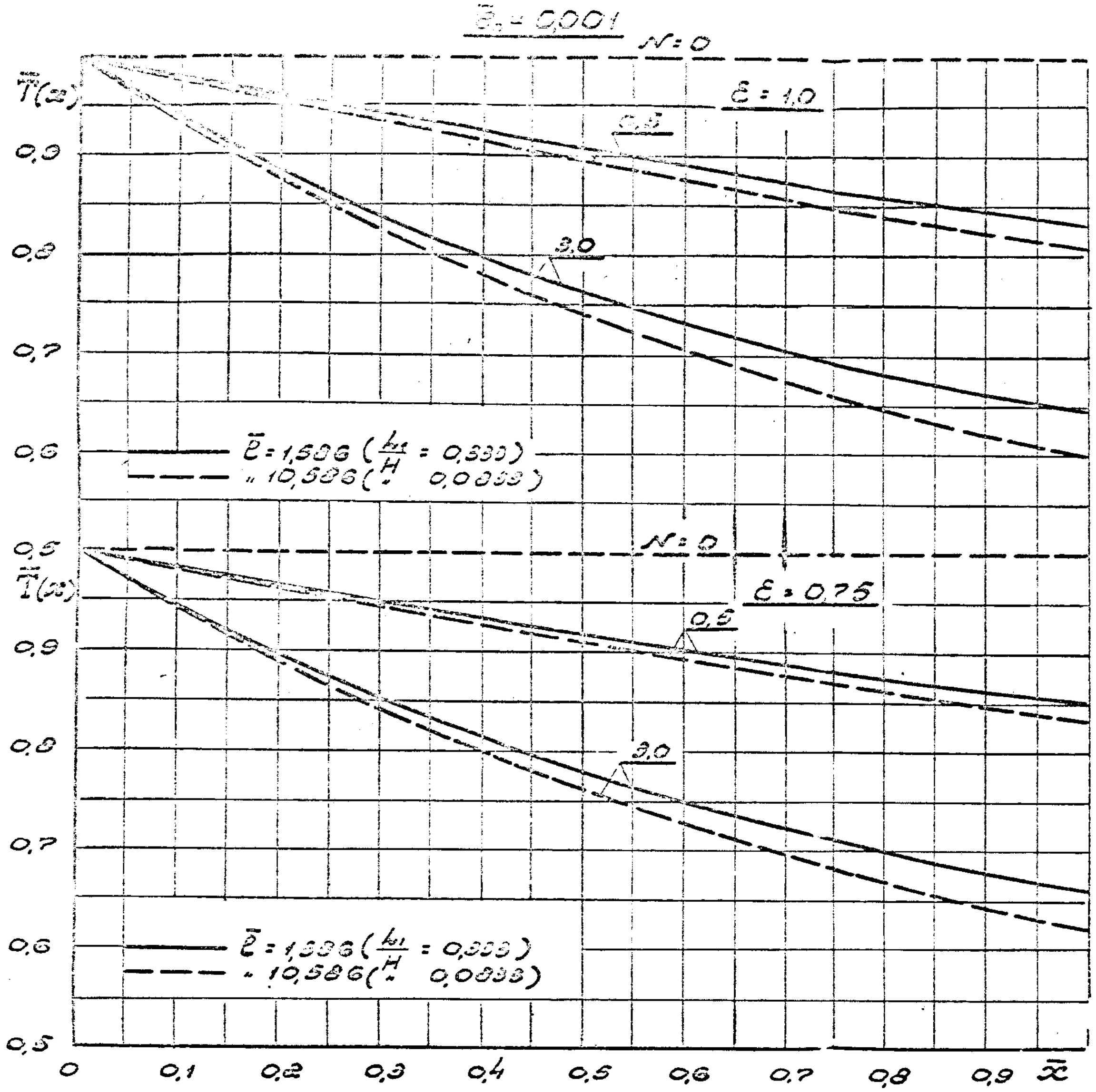


Figure 9

of selected values for ε_0, L, ℓ, T_0 and $\mathcal{E}$, the expression for determining the heat emitted by an individual emitter can be obtained from the following relationship:

$$q_\Sigma = \eta q_{id} = \eta\, 8 L_1 (1 + \bar{\varepsilon}_0) \sigma T_0^4 \sin 45^\circ \tag{48}$$

The system of equations obtained was solved numerically on an electronic

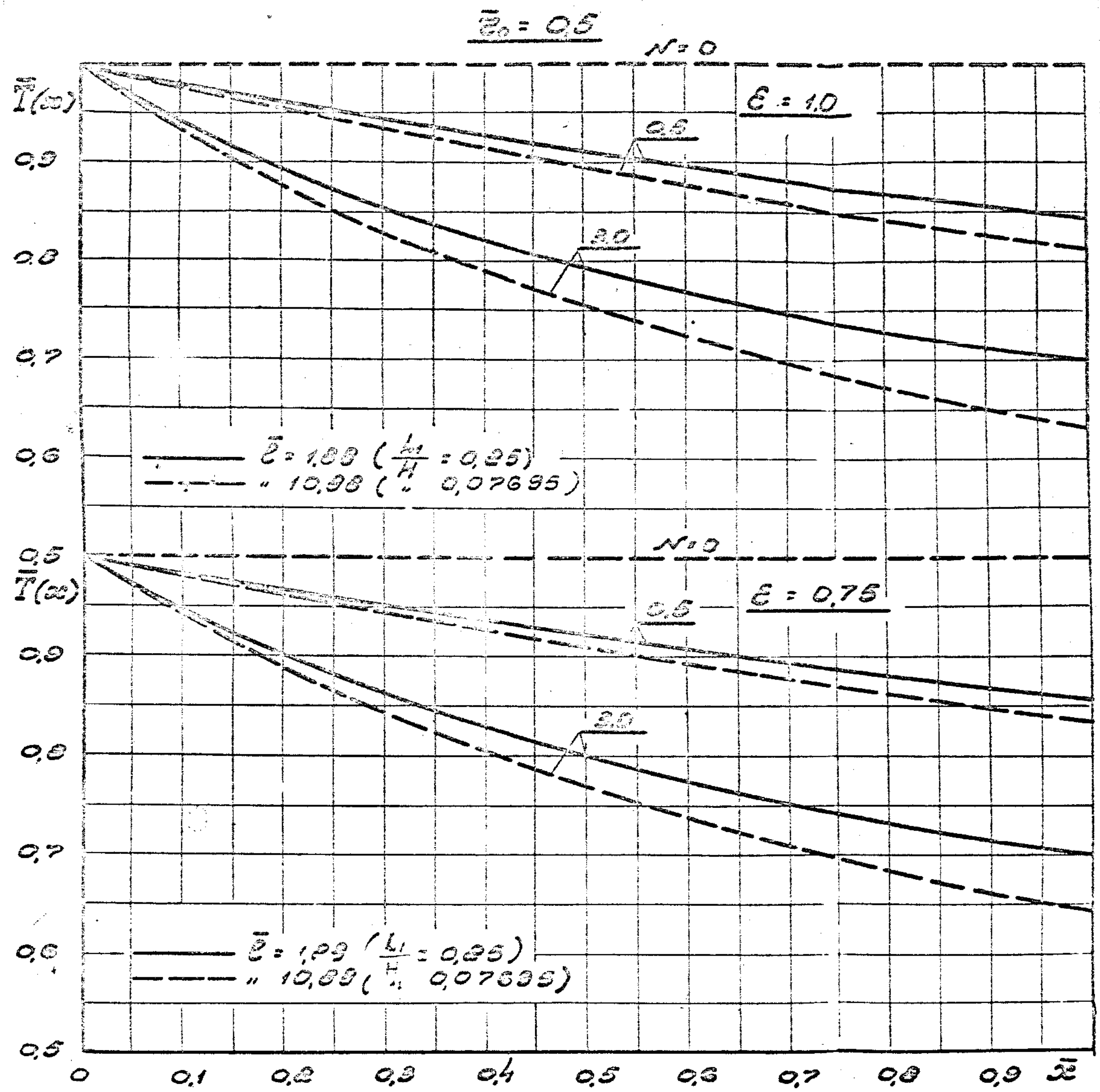


Figure 10

computer by the successive approximation method for $\bar{L} \rightarrow 1$, which corresponds to triangular fins.

The dependences $\eta = f(\frac{1}{\bar{H}})$, where $\bar{H} = \frac{H}{L_1} = 2(1+\bar{z}_0)\cos 45 + \bar{\ell}$; (see Figure 5a), are shown in Figures 6, 7 and 8 for different values of $\bar{z}_0$, $\mathcal{E}$ and $N = 2N_1$. A temperature change along the fin for several values of the parameters $\bar{z}_0, \mathcal{E}, \mathcal{N}$ and $\bar{\ell}$ is shown in Figures 9 and 10.

Utilizing the dependences shown in Figures 6 - 8, for a selected value of ε we can readily determine the geometry, optimum with respect to weight, of triangular fins of the emitters under consideration, if the prism side temperature T_0, the distance between the emitter axes H, the dimension of the prism ε_0, and the outgoing amount of heat q_Σ are known.

Actually, by employing the dependences shown in Figures 6 - 8 and expressions (41) and (47), for selected values of $\varepsilon, T_0, H, \varepsilon_0$ and q_Σ, we can determine the angles α corresponding to several values of L_1, and can thus find the fin geometry of a minimum area. /17

The dashed line in Figures 6 - 8 is used to plot the curves taken from (Ref. 4), which employed a similar formulation to study a system of star-shaped emitters, which is shown in Figure 5b. The points far to the right on the curves in Figures 6 - 8 correspond to the case $\bar{\ell}_1 = \frac{\ell_1}{L_1} = 0$ where ℓ_1 is the distance between the ends of horizontal fins of adjacent emitters of an emissive system. This is shown in Figure 5b. It can be seen that the system under consideration has somewhat worse emissivity as compared with the system in Figure 5b for small values of $\bar{H}$; however, in contrast to the latter, it has identical temperature distribution in all fins. With an increase in $\bar{H}$, the effectiveness of emitters in both systems coincides with the effectiveness of a single emitter (Ref. 5).

REFERENCES

1. Jacobi, M. "Problems of Heat Transfer". Izdatel'stvo Inostrannoy Literatury, 1960.

2. Psarouthakis, T. "Apparent Thermal Emissivity from Surfaces with Multiple V-Shaped Grooves". ATAA Journal, V. 1, No. 8, 1963.

3. Zhulev, Yu. G. "Emissivity of a Pinnacled Surface" (Izluchatel'naya sposobnost' zubchatoy poverkhnosti). Teplofizika Vysokikh Temperatur, No. 2, 1965.

4. Vasanov, Yu. A. "Theoretical Investigation of Thermal Emission of an Infinite System of Parallel, Star-Shaped Emitters" (Teoreticheskoye issledovaniye kharakteristik teplovogo izlucheniya beskonechnoy sistemy parallel'nykh zvezdoobraznykh izluchateley). Izvestiya AN SSSR, Energetika i Transport, No. 5, 1964.

5. Vasanov, Yu. A., Zhulev, Yu. G. "Optimum Form of Triangular Heat Transfer Fins with Allowance for Inter-Irradiation of Fins and of the Cooled Surface" (Optimal'naya forma treugol'nykh teplootvodyashchikh reber s uchetom vzaimnogo oblucheniya reber i okhlazhdayemoy poverkhnosti). Izvestiya AN SSSR, Energetika i Transport, No. 3, 1964.

Scientific Translation Service
4849 Tocaloma Lane
La Canada, California